FAR BEYOND

MAT122

Introduction to Limits

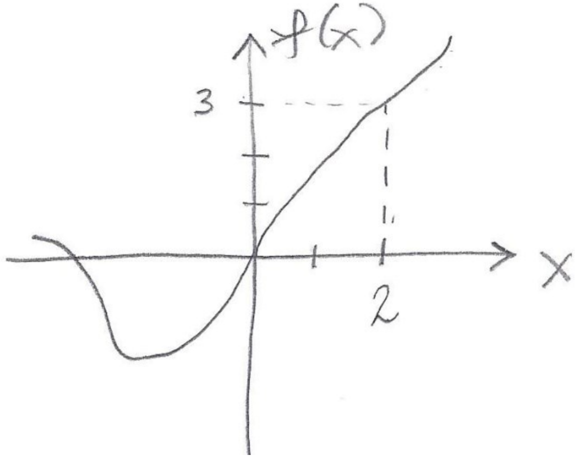
Stony Brook University

Limit of a Function

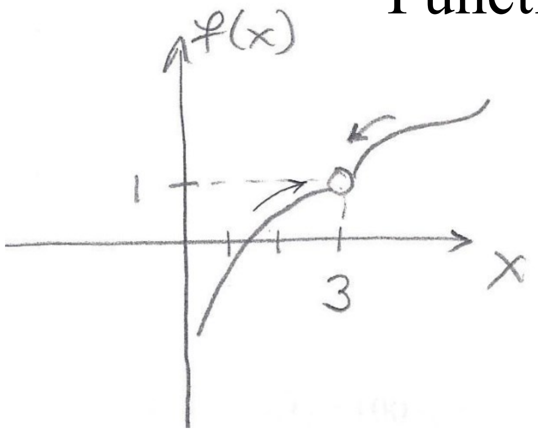
$$\lim_{x \to a} f(x) = L^*$$

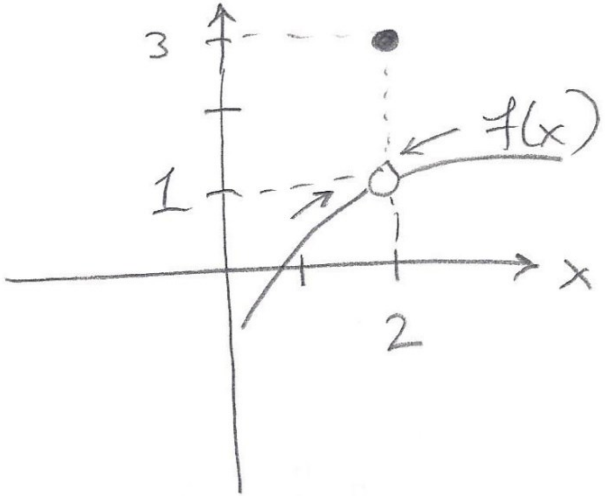
Determine Limit from a Graph

$$\lim_{x \to a} f(x) = L$$

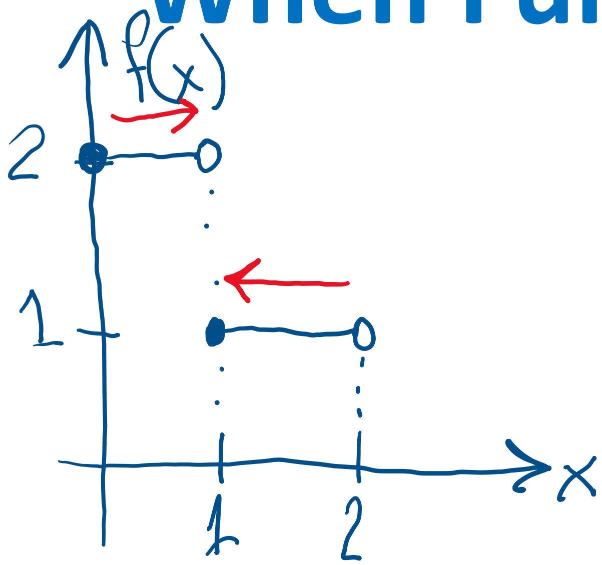


Function does not have to be defined at x-value to have a limit:





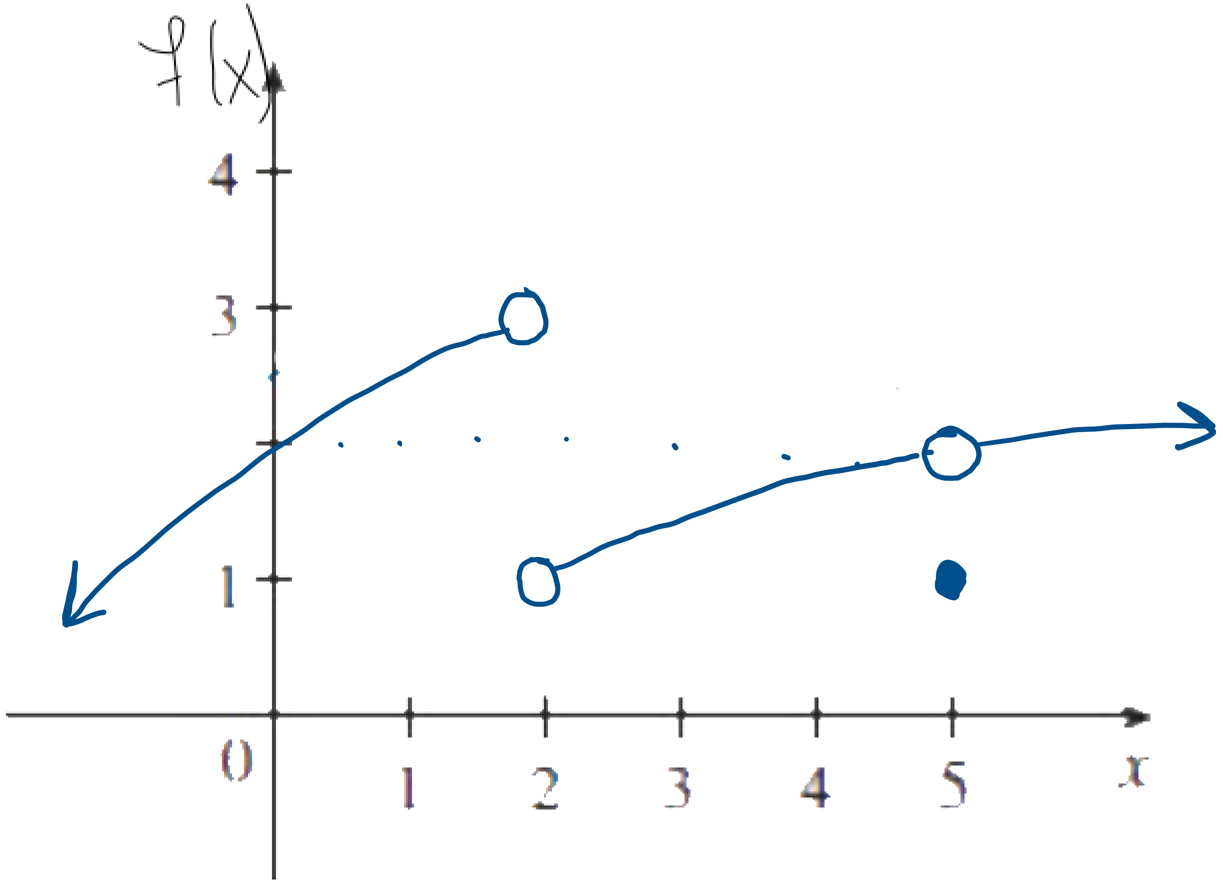
When Function Approaches Different *y*-values



$$\lim_{x \to a} f(x) = L \quad \text{iff both} \quad \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$

otherwise limit DNE

One-sided Limits from a Graph



ex. Evaluate the following limits:

$\lim_{x \to 2^-} f(x) =$

$\lim_{x \to 5^-} f(x) =$

$\lim_{x \to 2^+} f(x) =$

$\lim_{x \to 5^+} f(x) =$

$\lim_{x \to 2} f(x)$

$\lim_{x \to 5} f(x) =$

$f(2)$

$f(5) =$

$\lim_{x \to 0} f(x) =$

$f(0) =$

Evaluating Limits

ex. $\lim_{x \to 1} \left(3x^2 + 4x - 9 \right)$

Do: $\lim_{x \to 2} \dfrac{x^3 - 3}{x - 1}$

Limit Laws

Suppose the limits $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ both exist at $x = a$. Then:

$$\lim_{x\to a}\left(f(x)+g(x)\right) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left(f(x)-g(x)\right) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left(f(x)\cdot g(x)\right) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} \quad \text{assuming } g \neq 0$$

$$\lim_{x\to a} c = c \qquad \text{where } c \text{ is a constant}$$

$$\lim_{x\to a} c \cdot f(x) = c \lim_{x\to a} f(x)$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$$

Limit Laws - Practice

ex: Given: $\lim\limits_{x\to -3} f(x) = 5$ $\lim\limits_{x\to -3} g(x) = -4$ $\lim\limits_{x\to -3} h(x) = -1$

Use <u>Limit Laws</u> to calculate the following:

$ex. \lim\limits_{x\to -3} [2f(x) + 3g(x)]$

$$\boxed{\lim_{x\to a}\left(f(x)+g(x)\right) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)}$$

$$\boxed{\lim_{x\to a} c\cdot f(x) = c\lim_{x\to a} f(x)}$$

$ex. \lim\limits_{x\to -3} [g(x)]^3$

$ex. \lim\limits_{x\to -3} \sqrt{7 - h(x)}$

$$\boxed{\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}}$$

$$\boxed{\lim_{x\to a} c = c}$$